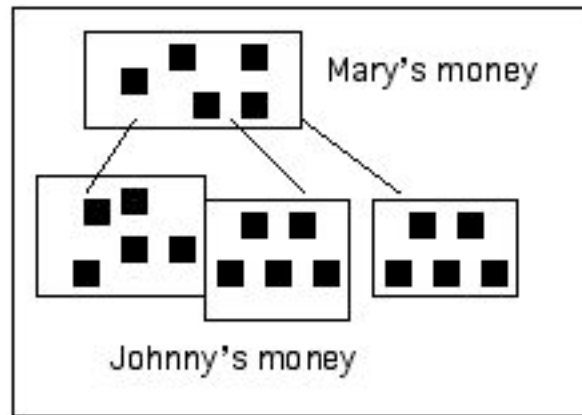


The comparison meaning of multiplication

You may have heard language such as “*I have twice as much as you.*”, “*My sister is five times as smart as you.*”, “*My car uses one-third of the gas yours does.*”, “*I lost 1/2 my money in the stock market yesterday.*”, “*I got an increase in pay of 10%.*”, etc. You may have seen telescopes or binoculars with symbols such as ‘x 15’ stamped on them. You may have seen maps with x100 indicated on them. These situations involve the comparison meaning of multiplication. Something is being compared to something else in a multiplication way. These situations can also be associated with ratio, a concept that concerns comparing two quantities.

Suppose Johnny has three times the money that Mary has. A diagram can be used to represent the situation.

Mary’s amount of money occurs three times in Johnny’s amount (there are three copies of Mary’s money in Johnny’s).



We speak of this kind of circumstance by saying something like; “*Johnny has three times as much money as Mary.*” Notice that the situation cannot be modeled well by division which concerns splitting stuff up into equal groups. While it is true that Johnny’s money is being split up into groups of 5 for purposes of finding out how many copies of Mary’s there are, this is not how we normally envision division. Division is seen as having a pile of stuff and splitting it all up into equal groups. Once split up, the groups stay that way. That is not what is going on in the Johnny and Mary situation. There are 20 pieces of money in the picture, not 15. Johnny still has one group of 15, not 3 groups of 5.

Note that we can also compare Mary's money to Johnny's money by writing the number sentence $1/3 \times 15 = 5$. For this, $1/3$ times Johnny's money equals Mary's money. Now the comparison number is '1/3'.

Another way to understand the comparison meaning of multiplication is as a shrink/magnify idea. If the multiplier is bigger than one, you are magnifying something. If the multiplier is less than one, you are shrinking something. This is the case for such situations as the magnifying power of binoculars and scales on maps.